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Journal
Nuclear Physics, Section B, 51(C)

ISSN
0550-3213

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Publication Date
1973

DOI
10.1016/0550-3213(73)90506-3

Nuclear Physics B51 (1973) 145–156. North-Holland Publishing Company

INCLUSIVE PRODUCTION FROM NUCLEAR TARGETS *

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Received 24 July 1972

Abstract: The inclusive production of a particles from a nuclear target is investigated. The cross section for such a process is related to a three body scattering amplitude. Once this amplitude is known for a nucleon target it may be found for a nuclear one by an extension of Glauber's multiple scattering formalism. A comparison with data is presented for proton induced inclusive production from various nuclear targets.

1. Introduction

Inclusive reactions of the form

$$a + b \to c + \ldots \tag{1}$$

continue to be of interest in high energy physics. Though the primary interest centers on the situation where both a and b are "simple" particles, i.e. mesons, nucleons and photons, experiments have been performed and it is inevitable that they will continue to be performed with "complicated" nuclear targets. Deuterium experiments will be useful for the determination of results on neutron initiated processes; out of necessity heavier nuclear targets will likewise be used. In this article we shall address ourselves to the question of how much information do we need on the process (1) with b either a proton or a neutron in order to determine the cross section for

$$a + A \to c + \ldots \;, \tag{2}$$

where A denotes a nucleus.

That the inclusive cross section for reaction (2) is not just a sum of cross sections of the type represented by (1) may be noted from the study of the simplest of the in-

* Supported in part by the National Science Foundation.
** Address from 1 September 1972 to December 15 1972: National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510, USA.

clusive processes, namely the total cross section. The total cross section, $\sigma(a + A)$, is *not* the sum of a + nucleon cross sections. At high energies, in order to determine the cross section off a nucleus one needs the elastic scattering amplitude off a nucleon, $f(q)$; in the situation that the nucleus is larger than the range of the scattering forces it is sufficient to know $f(0)$. Likewise for inclusive cross sections with nuclear targets effects of coherence, rescattering and absorption will be important.

It is usual [1] to relate the inclusive cross section for (1) to the forward three body amplitude for

$$a + b + \bar{c} \rightarrow a + b + \bar{c} \quad . \tag{3}$$

For our purpose it will be more convenient to relate the cross sections for processes (1) and (2) to discontinuities in the respective variables $E_a + E_b - E_c$ or $E_a + E_A - E_c$ of the reactions [2]

$$a + b + c \rightarrow a + b + c \quad , \tag{4a}$$

$$a + A + c \rightarrow a + A + c \tag{4b}$$

As in the case of the total cross sections a knowledge of the forward and nonforward, absorptive and dispersive amplitude for (4a) is necessary in order to be able to predict the cross section for (4b). From experiment such information will not be forthcoming for a long time. The cross section for reaction (1) gives us just the absorptive part of the forward amplitude for (4a). To find the other parts of the amplitude we shall have to rely on theoretical models. For many cases of present interest very reasonable approximations may be made which yield results only weakly sensitive to the differences in the models. One may turn the argument around and use the inclusive processes with nuclear targets to obtain some information on the nonforward parts of these three body amplitudes and have a larger amount of data to confront with theoretical models, as for example the Regge-Mueller [1] analysis.

In sect. 2 we shall discuss the relevant three to three amplitudes and an eikonal representation for them. Sect. 3 will be devoted to an extenstion of the Glauber [3] formalism to this three body situation. In sect. 4 we shall compare some of these predictions with experimental results of Allaby et al. [4]. These predictions are sensitive to the shape and content of the nuclear surface and thus may be a useful tool for studying it.

As we shall make extensive use of the eikonal approximation our results will be valid when neither $|\boldsymbol{p}_a - \boldsymbol{p}_c|$ nor $|\boldsymbol{p}_b - \boldsymbol{p}_c|$ are small.

2. Three particle amplitudes

2.1. *Relation of the inclusive cross section*

Let us consider reaction (4a) in the nonforward direction

$$p_a + p_b + p_c \to p'_a + p'_b + p'_c \ , \tag{5}$$

and let T_3 be its transition matrix related to the S-matrix by

$$S = 1 - i\, T_3\, \delta^4(p_a + p_b + p_c - p'_a - p'_b - p'_c) \ . \tag{6}$$

It is convenient to work with the analytic amplitude A_3 related to T_3 by

$$T_3 = -\frac{1}{\pi}\, \frac{1}{E_a E_b E_c}\, A_3 \ . \tag{7}$$

The inclusive cross section for reaction (1), $\mathrm{d}\sigma/\mathrm{d}p_c$, is then related to a discontinuity of A_3

$$\frac{p_{ab}\sqrt{S_{ab}}}{8\pi}\, E_c \frac{\mathrm{d}\sigma}{\mathrm{d}p_c} = [A_3(E_c + i\epsilon) - A_3(E_c - i\epsilon)]/2i \ , \tag{8}$$

with prime and unprimed variables being taken equal. p_{ab} and $\sqrt{S_{ab}}$ are the relative momentum and invariant mass of the a, b system in its c.m.

2.2. *Eikonal representation*

As in two body scattering we shall find the eikonal representation of three body amplitudes useful. At high energies multiple scattering is achieved as a product of transmision coeficients along lines of constant impact parameter.

Eikonal representations of three body states have been studied within a different context [5, 6[. We shall present a treatment useful for this work. A three particle to three particle amplitude is a function of eight variables. We shall group these into two sets. In the lab. system (particle b or A at rest) and a impinging along the Z axis, the first set consists of

$$E_a \simeq p_{a\,\parallel} \ , \qquad E_c \simeq p_{c\,\parallel} \ , \tag{9}$$

and of θ, the angle, or q^2 the relative transverse momentum, between a and c. For the second set we may choose $E'_a - E'_c$ and

$$\boldsymbol{q}_a = (\boldsymbol{p}'_a - \boldsymbol{p}_a)_\perp \ ,$$

$$\boldsymbol{q}_c = (\boldsymbol{p}'_c - \boldsymbol{p}_c)_\perp \ ; \tag{10}$$

$\boldsymbol{q}_a$ and $\boldsymbol{q}_c$ are two dimensional vectors transverse to the incident direction. It is convenient to introduce

$$\Delta = \tfrac{1}{2}[(E_a - E_c) - (E'_a - E'_c)] \ . \tag{11}$$

This choice will be useful for an eikonal representation under the restriction that θ (the angle between a and c) is small. For completeness, in the appendix we shall present the appropriate variables with this restriction removed.

The eikonal representation for a three particle amplitude may be inferred from a non-relativistic description of such a process:

$$T_3(E_{\mathrm{a}}, E_{\mathrm{c}} q^2; \boldsymbol{q}_{\mathrm{a}}, \boldsymbol{q}_{\mathrm{c}}, \Delta) \tag{12}$$
$$\simeq \int \mathrm{d}\boldsymbol{r}_{\mathrm{a}}\, \mathrm{d}\boldsymbol{r}_{\mathrm{c}}\, \mathrm{e}^{i(p'_{\mathrm{a}} - p_{\mathrm{a}}) \cdot r_{\mathrm{a}}}\, \mathrm{e}^{i(p'_{\mathrm{c}} - p_{\mathrm{c}}) \cdot r_{\mathrm{c}}}\, V(\boldsymbol{r}_{\mathrm{a}}, \boldsymbol{r}_{\mathrm{c}}; E_{\mathrm{a}}, E_{\mathrm{c}}, q^2) \ ,$$

with V some operator involving the potential and the full scattering wave function. Under the restriction of energy conservation and in the limit of high energies the exponential in (12) reduces to

$$\exp i[\boldsymbol{q}_{\mathrm{a}} \cdot \boldsymbol{b}_{\mathrm{a}} + \boldsymbol{q}_{\mathrm{c}} \cdot \boldsymbol{b}_{\mathrm{c}} + \Delta Z] \ , \tag{13}$$

with $\boldsymbol{b}$ representing the two dimensional position vector transverse to the Z direction and $Z = Z_{\mathrm{a}} - Z_{\mathrm{c}}$. With the above in mind the representation for the three body amplitude we shall use is

$$S(E_{\mathrm{a}}, E_{\mathrm{c}}, q^2; \boldsymbol{q}_{\mathrm{a}}, \boldsymbol{q}_{\mathrm{c}}, \Delta) = \delta(E_{\mathrm{a}} + E_{\mathrm{c}} - E'_{\mathrm{a}} - E'_{\mathrm{c}})$$
$$\times \frac{1}{(2\pi)^5} \int \mathrm{d}\boldsymbol{b}_{\mathrm{a}}\, \mathrm{d}\boldsymbol{b}_{\mathrm{c}}\, \mathrm{d}Z \exp i[\boldsymbol{q}_{\mathrm{a}} \cdot \boldsymbol{b}_{\mathrm{a}} + \boldsymbol{q}_{\mathrm{c}} \cdot \boldsymbol{b}_{\mathrm{c}} + \Delta Z]\, \hat{S}(E_{\mathrm{a}}, E_{\mathrm{c}}, q^2; \boldsymbol{b}_{\mathrm{a}}, \boldsymbol{b}_{\mathrm{c}}, Z) \ . \tag{14}$$

The crucial assumption we shall make is that the convolution due to multiple scattering are taken into account as a simple product of the appropriate $\hat{S}$'s.

2.3. *Specific form of the three particle amplitude*

Using potential theory as a guide we note that the three body $\hat{S}$ operator has the following relation to the interparticle potentials:

$$\hat{S}(b_{\mathrm{a}}, b_{\mathrm{c}}, Z) = \exp \left\{ i \int \mathrm{d}\left(\frac{Z_{\mathrm{a}} + Z_{\mathrm{c}}}{2} \right) [V_{\mathrm{ab}}(\boldsymbol{r}_{\mathrm{a}}) + V_{\mathrm{ac}}(\boldsymbol{r}_{\mathrm{a}} - \boldsymbol{r}_{\mathrm{c}}) \right.$$
$$\left. + V_{\mathrm{bc}}(\boldsymbol{r}_{\mathrm{c}}) + V_{\mathrm{abc}}(\boldsymbol{r}_{\mathrm{a}}, \boldsymbol{r}_{\mathrm{c}})] \right\} \ ; \tag{15}$$

V_{ij} represents a two body potential and V_{abc} is a genuine three body interaction. From (15) we may abstract a more general expression:

$$S(\boldsymbol{b}_{\mathrm{a}}, \boldsymbol{b}_{\mathrm{c}}, Z) = \hat{S}_{\mathrm{ac}}(\boldsymbol{b}_-)\hat{S}_{\mathrm{ab}}(\boldsymbol{b}_{\mathrm{a}})\, \hat{S}_{\mathrm{bc}}(\boldsymbol{b}_{\mathrm{c}})\, \hat{S}_{\mathrm{abc}}(\boldsymbol{b}_+, \boldsymbol{b}_-, Z) \ ; \tag{16}$$

$\hat{S}_{ij}$ is the impact parameter representation of two body elastic scattering and $\hat{S}_{\mathrm{abc}}$ is that part of the three body scattering matrix that is two body irreducible. Eq. (16) is represented diagramatically in fig. 1. In the above we have introduced the convenient variables

$$\boldsymbol{b}_- = \boldsymbol{b}_{\mathrm{a}} - \boldsymbol{b}_{\mathrm{c}} \ ,$$
$$\boldsymbol{b}_+ = \tfrac{1}{2}(\boldsymbol{b}_{\mathrm{a}} + \boldsymbol{b}_{\mathrm{c}}) \ . \tag{17}$$

The relation between $\hat{S}_{ij}$ and the two body scattering amplitude $f(q)$ is

$$\hat{S}_{ij}(b) = 1 + \Gamma_{ij}(b) \ ,$$
$$\Gamma_{ij}(b) = \frac{i}{2\pi p} \int \mathrm{d}q\, \mathrm{e}^{-iq \cdot b}\, f(q) \ . \tag{18}$$

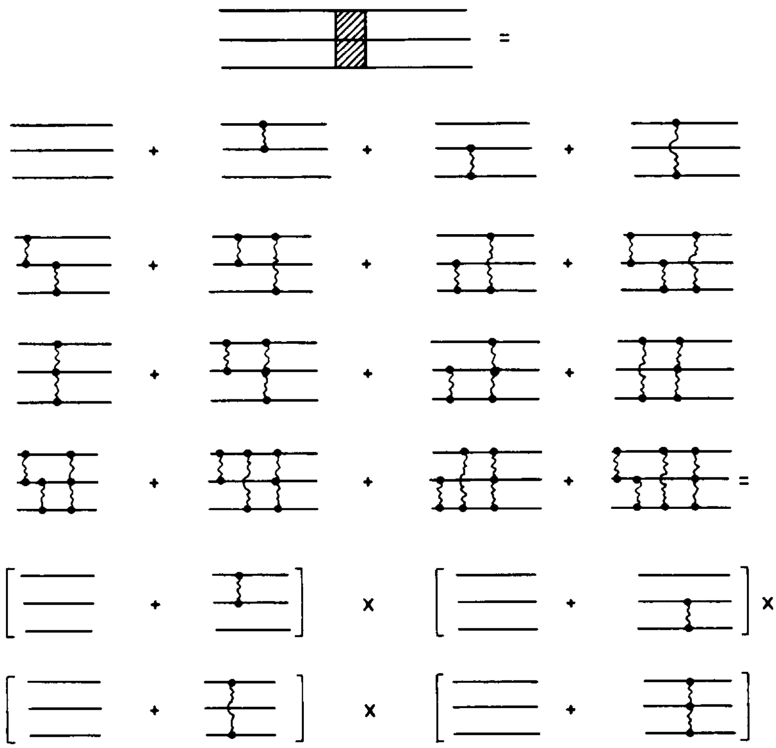

Fig.1. Diagramatic representation of eq. (16). Wavy lines indicate either a two body amplitude or two particle ireducible part of the three body amplitude.

An analogous separation of $\hat{S}_{\rm abc}$ is

$$\hat{S}_{\rm abc}(E_{\rm a}, E_{\rm c}, q^2; \boldsymbol{b}_+, \boldsymbol{b}_-, Z) = 1 + \frac{i}{8\pi^2 p_{\rm c}} \phi(E_{\rm a}, E_{\rm c} q^2; \boldsymbol{b}_+, \boldsymbol{b}_-, Z) \tag{19}$$

We assume that the Γ's and ϕ go to zero rapidly as the impact parameters or Z become large compared to the range of forces. Following the chain of arguments from eq. (8) we may relate the inclusive cross section to ϕ:

$$E_{\rm c} \frac{{\rm d}\sigma}{{\rm d}p_{\rm c}} = \frac{1}{(2\pi)^5} \int {\rm d}\boldsymbol{b}_+ \, {\rm d}\boldsymbol{b}_- \, {\rm d}Z \, \hat{S}_{\rm ac}(\boldsymbol{b}_-) \hat{S}_{\rm ab}(\boldsymbol{b}_{\rm a}) \hat{S}_{\rm cb}(\boldsymbol{b}_{\rm c})$$

$$\times \, [\phi(E_{\rm a}, E_{\rm c} + i\epsilon, \ldots) - \phi(E_{\rm a}, E_{\rm c} - i\epsilon, \ldots)] \ . \tag{20}$$

Aside from the above restriction we have no further experimental information about ϕ. Such information would have to come from genuine three body scatering. For the present we shall have to rely on theoretical models.

Some eminently reasonable assumptions may be made about the order of magnitude of ϕ, which will be crucial for the subsequent discussion. First we assume that the dispersive part of ϕ is of the same order of magnitude as its absorptive part. Second, we assume that all the important contributions to the integrand of eq. (20) occur when b_+, b_- and Z are of the order of the range of nuclear forces, r_0, and there are no large cancelation in the integration. With these assumptions and noting that for inclusive cross sections of interest $E_c\,d\sigma/dp_c$ is of the order or less than πr_0^4 we obtain a bound for ϕ:

$$\pi^2 r_0^4 > \phi\, r_0^5 \quad . \tag{21}$$

Parenthetically it may be noted that eq. (16) provides a mechanism for the inclusion of cuts into a Regge-Mueller [1] analysis. $\hat{S}_{abc}$ could be taken to be a simple Mueller diagram and the product would take into account effects of absorption.

3. Nuclear targets

3.1. *General solution*

Under the conditions of validity of the eikonal method, a knowledge of ϕ and the Γ's (eq. (18) and (19)) is sufficient to obtain the one particle inclusive spectrum off a nuclear target. Of course, we have to know the ground state nuclear wave function. For simplicity we assume that the nucleus is made up of one type of particle. A generalization to protons and neutrons is straightforward.

The procedure we follow is to first determine the forward amplitude for reaction (4b) and then take the discontinuity indicated in (8). Following Glauber [3] and the discussion of the previous section we find the impact parameter representation of the nuclear three body scattering matrix:

$$\hat{S}^{(A)}(\boldsymbol{b}_a, \boldsymbol{b}_c, Z) = \hat{S}_{ac}(\boldsymbol{b}_-)\langle G|\prod_{i=1}^{A}\hat{S}'(\boldsymbol{b}_a - \boldsymbol{c}_i, \boldsymbol{b}_c - \boldsymbol{c}_i, Z)|G\rangle \quad . \tag{22}$$

In the above

$$\hat{S}' = \hat{S}_{ab}\,\hat{S}_{cb}\,\hat{S}_{abc} \quad , \tag{23}$$

and the c_i are the transverse positions of the nuclei inside the nucleus. The matrix element is taken in the ground state $|G\rangle$, of the nucleus. In principle (22) is a complete solution of our problem. However, to be useful we must consider specific forms for the nuclear wave function. Two simple cases are presented below: the uncorrelated independent particle nucleus and the deuteron.

3.2. *Independent particle nucleons*

The square of the nuclear wave function is taken to have the form

$$|\psi(r_1, \ldots, r_A)|^2 = \prod_{i=1}^{A}\rho(r_i) \quad . \tag{24}$$

The densities, $\rho(r)$, are normalized to unity. As the nuclear radius increases as $A^{\frac{1}{3}}$, the ρ's decrease on A^{-1}. Letting $T(c)$ be the optical thickness at impact parameter c,

$$T(c) = A \int \rho(r_Z, c)\, \mathrm{d}r_Z \quad , \tag{25}$$

eq. (22) becomes

$$\hat{S}^{(A)}(b_a, b_c, Z) = \hat{S}_{ac}(b_-) \left[\int \mathrm{d}C \frac{T(c)}{A} \hat{S}'(b_+ - C, b_-, Z) \right]^A \quad , \tag{26}$$

or

$$\hat{S}^{(A)} = \hat{S}_{ac} \left[1 + \int \mathrm{d}c \frac{T}{A} (\hat{S}' - 1) \right]^A \quad . \tag{27}$$

If the nucleus is large compared to the ranges involved in the scattering forces (27) may be simplified to

$$\hat{S}^{(A)}(b_+, b_-, Z) = \hat{S}_{ac}(b_-) \left\{ 1 + \frac{T(b_+)}{A} \int \mathrm{d}b'_+ [\hat{S}'(b'_+, b_-, Z) - 1] \right\}^A \tag{28}$$

In the case of two body scattering this simplification manifests itself in the fact that we need know only the forward amplitude $f(0)$ rather than for arbitrary angles [3].

For large A we have a further convenient simplification [7],

$$\hat{S}^A(b_+, b_-, Z) = \hat{S}_{ac}(b_-) \exp \left\{ T(b_+) \int \mathrm{d}b'_+ [\hat{S}'(b'_+, b_-, Z) - 1] \right\} \tag{29}$$

To obtain the nuclear inclusive cross section we must integrate over b_+, b_- and Z and take the appropriate discontinuities. Unless we know the dependence on b_- and Z of the function ϕ (eq. (19)) we cannot perform these integrations explicitely. The bounds obtained in eq. (21) provide us with a useful approximation As long as

$$\frac{\phi R}{v p_c} << 1 \tag{30}$$

(R is the nuclear radius and v the volume per nucleon), we can make an expansion of the exponent in (29). Letting $b'_a = b'_+ + \frac{1}{2} b_-$ and $b'_c = b'_+ - \frac{1}{2} b_-$ this expansion yields

$$\hat{S}^{(A)}(b_+, b_-, Z) = \hat{S}_{ac}(b_-) \exp \left\{ T(b_+) \int \mathrm{d}b'_+ [\hat{S}_{ab}(b'_a)\, \hat{S}_{cb}(b'_c) - 1] \right\}$$

$$\times \left[1 + \frac{i\, T(b_+)}{8\, \pi^2 p_c} \int \mathrm{d}b'_+\, \phi(b'_+, b_-, Z)\, \hat{S}_{ab}(b'_a)\, \hat{S}_{ac}(b'_c) + \ldots \right] . \tag{31}$$

Using the estimate of (21) the condition necessary for the validity of the expansion is

$$R/(r_0^2 p_c) \ll 1 \quad . \tag{32}$$

Of course if R should be very large, then even for high energies we would have to take more terms. For the experimental comparison we have in mind the first term is sufficient.

For a variety of diffractive models of elastic scattering the integral appearing in the exponent of (31) may be evaluated and the result expressed as

$$\int \mathrm{d}b'_+ [\hat{S}_{\mathrm{ac}}(b'_{\mathrm{a}})\, \hat{S}_{\mathrm{cb}}(b'_{\mathrm{c}}) - 1]$$

$$= -\tfrac{1}{2}\sigma_{\mathrm{ab}} - \tfrac{1}{2}\sigma_{\mathrm{cb}} + \frac{\sigma_{\mathrm{ab}}\sigma_{\mathrm{cb}}}{4\pi r_0^2}\, f(b_-^2/r_0^2)\,, \tag{33}$$

with r_0 being the range of two body scattering and $f(x^2)$ is a function whose integral over x is one and whose range is unity.

It is the appearance of b_- in (33) that prevents the integration of the discontinuity of (31) in terms of nucleon inclusive cross sections. In practice the term involving b_- in (33) is small compared to the other two terms and thus we may consider two limiting cases. These situations depend on whether the range, in b_-, of the connected part of the three body amplitudes, ϕ, is shorter or much longer than the range of the two body amplitudes. In the first situation (short range) the function $f(x)$ of (33) can be replaced by unity and the nuclear inclusive cross section becomes

$$\frac{\mathrm{d}\sigma^{(A)}}{\mathrm{d}p_{\mathrm{c}}} = \frac{\mathrm{d}\sigma}{\mathrm{d}p_{\mathrm{c}}} \int \mathrm{d}\boldsymbol{b}_+\, T(\boldsymbol{b}_+) \exp\left\{-\tfrac{1}{2}T(\boldsymbol{b}_+)\left[\sigma_{\mathrm{ab}} + \sigma_{\mathrm{cb}} - \frac{\sigma_{\mathrm{ab}}\sigma_{\mathrm{cb}}}{2\pi r^2}\right]\right\}\,. \tag{34}$$

In the other situation (long range) the function $f(x)$ is non-zero for a small portion of the b_- integration and thus may be neglected. The resulting expression is

$$\frac{\mathrm{d}\sigma^{(A)}}{\mathrm{d}p_{\mathrm{c}}} = \frac{\mathrm{d}\sigma}{\mathrm{d}p_{\mathrm{c}}} \int \mathrm{d}\boldsymbol{b}_+\, T(\boldsymbol{b}_+) \exp\, \{-\tfrac{1}{2}T(\boldsymbol{b}_+)\, [\sigma_{\mathrm{ab}} + \sigma_{\mathrm{cb}}]\} \tag{35}$$

It is fortunate that the two expressions (34) and (35) do not give too different answers for cases of present practical interest.

It should be noted that even in the approximation of retaining the production term only once, (34) and (35) are not just superpositions of individual production amplitudes.

3.3. *Deuteron target*

Without going into details analogous to those of the previous section the inclusive production off a deuteron target is

(i) Short range:

$$\frac{\mathrm{d}\sigma^{(\mathrm{D})}}{\mathrm{d}p_{\mathrm{c}}} = \frac{\mathrm{d}\sigma^{(\mathrm{p})}}{\mathrm{d}p_{\mathrm{c}}}\left[1 + \pi\left(\sigma_{\mathrm{ap}} + \sigma_{\mathrm{cp}} - \frac{\sigma_{\mathrm{ap}}\sigma_{\mathrm{cp}}}{2\pi r_{\mathrm{D}}^2}\right)\left\langle \frac{1}{r_{\mathrm{D}}^2}\right\rangle\right]$$

$$+ (\mathrm{p} \to \mathrm{n})\ ; \tag{36}$$

(ii) Long range:

$$\frac{\mathrm{d}\sigma^{(\mathrm{D})}}{\mathrm{d}p_\mathrm{c}} = \frac{\mathrm{d}\sigma^{(\mathrm{p})}}{\mathrm{d}p_\mathrm{c}} \left[1 + \pi(\sigma_{\mathrm{ap}} + \sigma_{\mathrm{cp}}) \langle \frac{1}{r_\mathrm{D}^2} \rangle \right] + (\mathrm{p} \to \mathrm{n}) \quad ; \tag{37}$$

r_D is the deuteron radius.

4. Comparison with proton induced reactions

The data to which we shall compare the previous theory to is on inclusive production of various particles by 19.2 GeV/c incident protons on Be, Al, Cu and Pb [4]. As may be noted from (34) and (35) the ratio on nucleus to nucleon cross section is dependent on the shape and composition of the nuclear rim. Three different forms of the nuclear density were considered.

4.1. *Nuclear densities*

(i) Density 1: Saxon-Wood model. This is the canonical nuclear density valid for heavier nuclei [8],

$$\rho(r) = \rho(0)/[1 + \exp((r - R)/d)] \ , \tag{38}$$

with R the nuclear radius and d the skin depth. The values used were $R = 1.12\ A^{\frac{1}{3}}$ fm and $d = 0.545$ fm.

(ii) Density 2: diffuse neutron boundary. There is evidence that the neutron surface extends out beyond the protons [9]. The proton density is taken to be as in (38) while the neutron skin depth is taken to be $d = 1.21$ fm.

(iii) Density 3: Gaussian distribution. For lighter nuclei a shell model density should be used,

$$\rho(r) = \rho(0) \exp(-r^2/R^2) \tag{39}$$

4.2. *Results*

A detailed comparison with data is presented for the ratioıof nuclear target to proton target cross sections for the production of π^+, π^- and K^+ at 12.5 mrad and various outgoing momenta p_c. The experimental results and predictions of (34) and (35) for the various nuclear densities discussed above are presented on table 1 and table 2. As the difference which resulted in the variation of σ_{bc} with energy was negligible, the $\mathrm{K}^+\mathrm{p}$ cross section was set at 17 mb and the $\pi^\pm\mathrm{p}$ one at 27 mb.

For K^+ and π^- production we get good agreement with calculations using density 2, while for π^+ production the theoretical ratio tends to be higher. We do not get the marked fall off with increasing p_c. In this calculation we have neglected any difference in production off protons or neutrons. For π^+ production the reaction

Table 1

Ratio of inclusive cross section for p + A → $\pi^\pm$ + . . . to p + p → $\pi^\pm$ + . . . for 19.2 GeV/c protons at 12.5 mrad. The definitions of terms used in the theoretical calculation are made in the text. The experimental data are from ref. [4] and the errors in the experimental ratio are 6%. p_c is in GeV/c.

			Short range			Long range		
Target	p_c	Experiment	Density 1	Density 2	Density 3	Density 1	Density 2	Density 3
		π^+/π^-						
	6	4.35/5.6						
	8	4.45/5.4						
Be	10	4.5/5.2	4.6	5.5	5.7	3.8	4.9	5.0
	12	4.35/4.65						
	14	3.6/4.45						
	16	3.0/						
	6	7.4/10.6						
	8	8.3/10.3						
	10	8.3/9.5						
A_1	12	7.3/7.7	8.1	11.6	15.2	6.1	9.5	12.9
	14	6.0/7.4						
	16	4.6/						
	6	12.6/17.5						
	8	13.0/16.6						
	10	12.9/14.5						
Cu	12	11.3/12.1	11.0	18.0	30.5	7.9	14.1	24.9
	14	9.3/11.7						
	16	7.1/						
	6	20.6/29.0						
	8	20.6/27.1						
	10	19.8/22.6						
Pb	12	15.7/19.9	17.2	28.2	73.5	10.5	21.2	56.3
	14	14.0/17.9						
	16	10.9/						

Table 2
Ratio of inclusive cross section for $p + A \to K^+ + \ldots$ to $p + p \to K^+ + \ldots$ for 19.2 GeV/c protons at 12.5 mrad. The definitions of terms used in the theoretical calculation are made in the text. The experimental data are from ref. [4] and the errors in the experimental ratio are 6% to 7%. p_c is in GeV/c.

			Short range			Long range		
Target	P_c	Experiment	density 1	density 2	density 3	density 1	density 2	density 3
	6	5.2						
	8	4.8						
Be	10	4.65	4.8	5.7	5.9	4.3	5.2	5.4
	12	4.4						
	14	4.35						
	16							
	6	11.5						
	8	10.3						
Al	10	8.65	8.7	12.2	15.9	7.3	10.7	14.3
	12	8.45						
	14	7.9						
	16							
	6	20.5						
	8	16.8						
Cu	10	14.5	12.2	19.4	32.2	9.7	16.3	28.3
	12	13.2						
	14	14.1						
	16							
	6	35.2						
	8	28.0						
Pb	10	23.8	17.2	30.85	79.1	13.1	25.1	60.5
	12	20.0						
	14	20.0						
	16							

$p + p \to \pi^+ + d$ has no analogue on a neutron target. This reaction would tend to decrease the experimental ratio of nucleus to nuclear cross section for π^+ production, especially at higher values of p_c. For production of K^+ or π^- no such mechanism comes into play. (The reaction $pn \to \pi^- + d$ is not likely to yield a fast π^-.)

It is gratifying to note that the sensitivity to the range of the three body forces is sufficiently mild to permit this rough comparison with data.

For bringing this problem to may attention and many subsequent discussions I wish to thank Dr. D. Silverman.

Appendix

The choice of variables discussed in subsect. 2.2 is suitable for the case when the angle between p_a and p_c is small. For completeness we present a generalization valid for all angles. The difference consists of redefining the transverse and longitudinal directions for particle c to be with respect to p_c and not p_a. b_a, q_a and Z_a remain as before, while b_c, q_c are the position and momentum transfer vectors normal to p_c and Z_c is the postion along p_c. Δ and Z remain as before.

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